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AQA, OCR, Edexcel

GCSE

GCSE Maths

Proof Answers

Name:

MME

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Total Marks: /37

Proof

1. The n[th] even number is $2n$.
   a. The next even number can be written as $2n + 2$
   Explain why

- n is any number, $2n$ must be even as it is divisible by 2
- The next number is $2n + 1$ which is odd

   b. Write down an expression, in terms of n, for the next even number after $2n + 2$.

- $2n + 4$

   c. Show algebraically that the sum of any 3 consecutive even numbers is always a divisible by 6

- $2n + (2n + 2) + (2n + 4) =$
- $6n + 6$
- $6(n + 1)$

(3 Marks)

2. Prove, using algebra, that the sum of two consecutive integers is always odd.

- $n + (n + 1)$
- $= 2n + 1$
- $2n$ is always even so $2n + 1$ is always odd

(2 Marks)

3. Prove algebraically that $(4n + 2)^2 - (2n + 2)^2$ is a divisible by 4 for all positive integers.
   - $(16n^2 + 16n + 4) - (4n^2 + 8n + 4)$
   - $12n^2 + 8n$
   - $4n(3n + 2)$

(4 Marks)

4. Prove that $(2n+3)^2 - (2n-3)^2$ is a multiple of 8 for all positive integers of n.

- $(2n+3)(2n+3) - (2n-3)(2n-3)$
- $(4n^2+12n+9) - (4n^2-12n+9)$
- $24n = 8(3n)$

(3 Marks)

5. Prove algebraically that $(3n+1)(n+3) - n(3n+7) = 3(n+1)$

- $(3n^2+10n+3) - (3n^2+7n)$
- $3n+3 = 3(n+1)$

(3 Marks)

6. Prove Algebraically that $\frac{1}{8}(4n+1)(n+8) - \frac{1}{8}n(4n+1) = 4n+1$

$$\frac{1}{8}(4n^2+33n+8) - \frac{1}{8}(4n^2+n)$$

$$\frac{(4n^2+33n+8)-(4n^2+n)}{8}$$

$$\frac{32n+8}{8}$$

RHS

(4 Marks)

7. Prove algebraically that the sum of two consecutive square numbers is twice the product of two consecutive numbers +1.

$n^2 + (n+1)^2 = n^2 + n^2 + 2n + 1 = 2n^2 + 2n + 1$

$= 2(n^2+n) + 1$

$= [2 \times n(n+1)] + 1$

(4 Marks)

8. Prove algebraically that the sum of 4 consecutive square numbers is divisible by 4 remainder 2.

(5 Marks)

$n^2 + (n+1)^2 + (n+2)^2 + (n+3)^2$

$4n^2 + 12n + 14 = (4n^2 + 12n + 12) + 2$

$4(n^2 + 3n + 3) + 2$

9. Show that the difference between 14^{20} and 21^2 is a multiple of 7.

$14^{20} - 21^2 = (2 \times 7)^{20} - (3 \times 7)^2$

$= 7^{20}2^{20} - 3^2 7^2$

$= 7(7^{19}2^{20} - 3^2 7)$

(3 Marks)

10. Tom says that $7x - (2x+3)(x+2)$ is always negative. Is he correct? Explain your answer.

$$Yes, \quad 7x - (2x+3)(x+2) = -2x^2 - 6 < 0$$

Yes, the expression is always negative. The square of any real number is always positive. Since we have $-2x^2$ we know that is always negative, irrespective of the value of x. Thus, the expression is less than zero for every real x.

(3 Marks)

11. Show that $3^{60} - 25$ is not a prime.

$$3^{60} - 25 = 3^{60} - 5^2 = (3^{30} - 5)(3^{30} + 5)$$

So it cannot be prime as we can express it as the product of two factors.

(3 Marks)